

Substitution Method

Let the variables be x , y , and z . The equations are eq.1, eq.2, and eq.3. The substitution method has these basic steps:

1. Solve eq.1 for z .
2. Substitute the whole right-hand side of the equation (the one you just wrote) in place of "z" in eq.2. You should use parentheses around the substituted expression to keep everything orderly.
3. Solve the equation you wrote in step 2 for y .
4. Substitute the whole right-hand side of the equation you wrote in step 1 in place of "z" in eq.3. Then substitute the whole right-hand side of the equation you wrote in step 3 in place of "y" in eq.3.
5. Solve the equation you wrote in step 4 for x . You should get a numeric value of x .
6. Use this value of x in the equation you wrote in step 3 to solve for y . You should get a numeric value of y .
7. Use these values of x and y in the equation you wrote in step 1 to solve for z . You should get a numeric value of z .

Example: three variables and three unknowns

$$3x - z + 120 = y$$

$$y - 2z = 30$$

$$x + y + z = 180$$

Step 1: Solve eq.1 for z .

$$3x - z + 120 = y$$

$$z = 3x + 120 - y$$

Step 2: Substitute z in eq.2

$$y - 2z = 30$$

$$y - 2(3x + 120 - y) = 30$$

Step 3: Solve the equation you wrote in step 2 for y .

$$y - 2(3x + 120 - y) = 30$$

$$y - 6x - 240 + 2y = 30$$

$$3y - 6x = 270$$

$$y - 2x = 90$$

$$y = 90 + 2x$$

Step 4: Substitute z , then substitute y in eq.3.

$$x + y + z = 180$$

$$x + y + (3x + 120 - y) = 180$$

$$x + (90 + 2x) + (3x + 120 - (90 + 2x)) = 180$$

Step 5: Solve the equation you wrote in step 4 to get the value of x.

$$x+(90+2x)+(3x+120-(90+2x))=180$$

$$x+90+2x+3x+120-90-2x=180$$

$$4x+120=180$$

$$x=15$$

Step 6: Use this value of x in the equation you wrote in step 3 to solve for y.

$$y=90+2x$$

$$y=90+2(15)$$

$$y=120$$

Step 7: Use these values of x and y in the equation you wrote in step 1 to solve for z.

$$z=3x+120-y$$

$$z=3(15)+120-120$$

$$z=45+120-120$$

$$z=45$$

Now you should check your answers in the original equations.

$$3x-z+120=y$$

$$3(15)-45+120=120$$

$$45-45+120=120$$

$$120=120$$

True.

$$y-2z=30$$

$$120-2(45)=30$$

$$120-90=30$$

$$30=30$$

True.

$$x+y+z=180$$

$$15+120+45=180$$

$$180=180$$

True.

You should be able to see how this generalizes to n equations in n unknowns. You would solve the first equation for the first unknown, then substitute this in the second equation, and solve for the second unknown. Then substitute both variables in the third equation and solve for the third unknown. Then substitute all three variables in the fourth equation, and solve for the fourth unknown, etc. When you get to the last equation, and solve it for the last unknown, you should be down

to a numeric answer. Then work backwards through the odd-numbered steps, getting numeric answers for each variable in turn.