

Use the Substitution Method to solve equations with three variables.

Example 1:

$$[1] \quad -x - y - 3z = -9$$

$$[2] \quad z = -3x - 1$$

$$[3] \quad x = 5y - z + 23$$

Equations Simplified so they all match

$$[1] \quad -x - y - 3z = -9$$

$$[2] \quad 3x + z = -1$$

$$[3] \quad x - 5y + z = 23$$

Solve by Substitution

// Solve equation [3] for the variable z

$$[3] \quad z = -x + 5y + 23$$

// Plug this in for variable z in equation [1]

$$[1] \quad -x - y - 3(-x + 5y + 23) = -9$$

$$[1] \quad 2x - 16y = 60$$

// Plug this in for variable z in equation [2]

$$[2] \quad 3x + (-x + 5y + 23) = -1$$

// Solve equation [2] for the variable y

$$[2] \quad 5y = -2x - 24$$

$$[2] \quad y = -2x/5 - 24/5$$

// Plug this in for variable y in equation [1]

$$[1] \quad 2x - 16(-2x/5 - 24/5) = 60$$

$$[1] \quad 42x/5 = -84/5$$

$$[1] \quad 42x = -84$$

// Solve equation [1] for the variable x

$$[1] \quad 42x = -84$$

$$[1] \quad x = -2$$

// By now we know this

$$x = -2$$

$$y = -2x/5 - 24/5$$

$$z = -x + 5y + 23$$

// Use the x value to solve for y

$$y = -(2/5)(-2) - 24/5 = -4$$

// Use the x and y values to solve for z

$$z = -(-2) + 5(-4) + 23 = 5$$

Solution

$$\{x, y, z\} = \{-2, -4, 5\}$$

Questions 2:

$$[1] \quad x = -4z - 19$$

$$[2] \quad y = 5x + z - 4$$

$$[3] \quad -5y - z = 25$$

Equations Simplified

$$[1] \quad x + 4z = -19$$

$$[2] \quad -5x - z + y = -4$$

$$[3] \quad -z - 5y = 25$$

Solve by Substitution

// Solve equation [2] for the variable y

$$[2] \quad y = 5x + z - 4$$

// Plug this in for variable y in equation [3]

$$[3] \quad -z - 5(5x + z - 4) = 25$$

$$[3] \quad -6z = 5$$

// Solve equation [1] for the variable x

$$[1] \quad x = -4z - 19$$

// Plug this in for variable x in equation [3]

$$[3] \quad -6z = 5$$

$$[3] \quad 94z = -470$$

// Solve equation [3] for the variable z

$$[3] \quad 94z = -470$$

$$[3] \quad z = -5$$

// By now we know this

$$x = -4z - 19$$

$$z = -5$$

$$y = 5x + z - 4$$

// Use the z value to solve for x

$$x = -4(-5) - 19 = 1$$

// Use the x and z values to solve for y

$$y = 5(1) + (-5) - 4 = -4$$

Solution

$$\{x, z, y\} = \{1, -5, -4\}$$

Question 3:

$$[1] \quad y = x + z + 5$$

$$[2] \quad z = -3y - 3$$

$$[3] \quad 2x - y = -4$$

Equations Simplified

$$[1] \quad y - x - z = 5$$

$$[2] \quad 3y + z = -3$$

$$[3] \quad -y + 2x = -4$$

Solve by Substitution

// Solve equation [2] for the variable z

$$[2] \quad z = -3y - 3$$

// Plug this in for variable z in equation [1]

$$[1] \quad y - x - (-3y - 3) = 5$$

$$[1] \quad 4y - x = 2$$

// Solve equation [3] for the variable y

$$[3] \quad y = 2x + 4$$

// Plug this in for variable y in equation [1]

$$[1] \quad 4 \cdot (2x + 4) - x = 2$$

$$[1] \quad 7x = -14$$

// Solve equation [1] for the variable x

$$[1] \quad 7x = -14$$

$$[1] \quad x = -2$$

// By now we know this

$$y = 2x + 4$$

$$x = -2$$

$$z = -3y - 3$$

// Use the x value to solve for y

$$y = 2(-2) + 4 = 0$$

// Use the y value to solve for z

$$z = -3(0) - 3 = -3$$

Solution

$$\{y, x, z\} = \{0, -2, -3\}$$

Question 4:

$$[1] \quad -2y + 5z = -3$$

$$[2] \quad y = -5x - 4z - 5$$

$$[3] \quad x = 4z + 4$$

Equations Simplified

$$[1] \quad -2y + 5z = -3$$

$$[2] \quad y + 4z + 5x = -5$$

$$[3] \quad -4z + x = 4$$

Solve by Substitution

// Solve equation [3] for the variable x

$$[3] \quad x = 4z + 4$$

// Plug this in for variable x in equation [2]

$$[2] \quad y + 4z + 5 \cdot (4z + 4) = -5$$

$$[2] \quad y + 24z = -25$$

// Solve equation [2] for the variable y

$$[2] \quad y = -24z - 25$$

// Plug this in for variable y in equation [1]

$$[1] \quad -2(-24z - 25) + 5z = -3$$

$$[1] \quad 53z = -53$$

// Solve equation [1] for the variable z

$$[1] \quad 53z = -53$$

$$[1] \quad z = -1$$

// By now we know this

$$y = -24z - 25$$

$$z = -1$$

$$x = 4z + 4$$

// Use the z value to solve for y

$$y = -24(-1) - 25 = -1$$

// Use the z value to solve for x

$$x = 4(-1) + 4 = 0$$

Solution

$$\{y, z, x\} = \{-1, -1, 0\}$$