

## Solving a Linear System in Three Variables

Use elimination to solve the following system of equations.

$$\begin{cases} 2x - y + 3z = 4 & \textcircled{1} \\ x + y - 2z = -7 & \textcircled{2} \\ 3x + 2y - z = -7 & \textcircled{3} \end{cases}$$

**Step 1** Eliminate one variable.

The coefficients of  $y$  are opposites in the first two equations.

$$\begin{array}{r} \textcircled{1} \quad 2x - y + 3z = 4 \\ \textcircled{2} \quad + x + y - 2z = -7 \\ \hline \quad 3x \quad \quad + z = -3 \end{array} \quad \begin{array}{l} \textit{Add equations } \textcircled{1} \\ \textit{and } \textcircled{2}. \end{array}$$

Use equations  $\textcircled{2}$  and  $\textcircled{3}$  to create a second equation in  $x$  and  $z$ .

$$\begin{array}{r} \textcircled{2} \quad 2(x + y - 2z = -7) \rightarrow 2x + 2y - 4z = -14 \quad \textcircled{4} \\ \textcircled{3} \quad - (3x + 2y - z = -7) \\ \hline \quad -x \quad \quad - 3z = -7 \\ \quad \quad \quad x + 3z = 7 \end{array} \quad \begin{array}{l} \textit{Multiply equation } \textcircled{2} \\ \textit{by 2. Subtract} \\ \textit{equation } \textcircled{4} \textit{ and } \textcircled{3}. \\ \textit{Multiply by } -1. \end{array}$$

You now have a 2-by-2 system. 
$$\begin{cases} 3x + z = -3 \quad * \\ x + 3z = 7 \end{cases}$$

**Step 2** Eliminate another variable.

$$\begin{array}{r} * \quad 3(3x + z = -3) \rightarrow 9x + 3z = -9 \\ \quad - (x + 3z = 7) \\ \hline \quad 8x \quad \quad = -16 \\ \quad \quad \quad x = -2 \end{array} \quad \begin{array}{l} \textit{Multiply equation } * \\ \textit{by 3 and subtract.} \\ \\ \textit{Solve for } x. \end{array}$$

**Step 3** Use one of the equations in the 2-by-2 system to solve for  $z$ .

$$\begin{array}{r} x + 3z = 7 \\ -2 + 3z = 7 \quad \textit{Substitute } -2 \textit{ for } x. \\ 3z = 9 \quad \textit{Add 2 to both sides.} \\ z = 3 \quad \textit{Divide both sides by 3.} \end{array}$$

**Step 4** Substitute for  $x$  and  $z$  in one of the original equations to solve for  $y$ .

$$\begin{array}{r} \textcircled{2} \quad x + y - 2z = -7 \\ \quad -2 + y - 6 = -7 \quad \textit{Substitute } -2 \textit{ for } x \textit{ and } 3 \\ \quad \quad y - 8 = -7 \quad \textit{for } z, \textit{ then simplify.} \\ \quad \quad y = 1 \quad \textit{Solve for } y. \end{array}$$