

John inherited \$25,000 and invested part of it in a money market account, part in municipal bonds, and part in a mutual fund. After one year, he received a total of \$1,620 in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the mutual fund paid 8% annually. There was \$6,000 more invested in the bonds than the mutual funds. Find the amount John invested in each category.

There are three unknowns:

- 1 : The amount of money invested in the money market account.
- 2 : The amount of money invested in municipal bonds.
- 3 : The amount of money invested in a mutual fund.

Let's rewrite the paragraph that asks the question we are to answer.

[The amount of money invested in the money market account + [The amount of money invested in municipal bonds ] + [The amount of money invested in a mutual fund ] = **\$25,000**.

The 6% interest on [ The amount of money invested in the money market account ]+ the 7% interest on [ The amount of money invested in municipal bonds ] + the 8% interest on [ The amount of money invested in a mutual fund ] = **\$1,620**

[The amount of money invested in municipal bonds ] - [ The amount of money invested in a mutual fund ] = **\$6,000**

It is going to get boring if we keep repeating the phrases

- 1 : The amount of money invested in the money market account.
- 2 : The amount of money invested in municipal bonds.
- 3 : The amount of money invested in a mutual fund.

Let's create a shortcut by letting symbols represent these phrases. Let

- $x$  = The amount of money invested in the money market account.  
 $y$  = The amount of money invested in municipal bonds.  
 $z$  = The amount of money invested in a mutual fund.

in the three sentences, and then rewrite them.

The sentence [ The amount of money invested in the money market account ] + [ The amount of money invested in municipal bonds ] + [ The amount of money invested in a mutual fund ] = **\$25,000**. can now be written as

$$x + y + z = \$25,000$$

The sentence The **6%** interest on [ The amount of money invested in the money market account ] + the 7% interest on [ The amount of money invested in municipal bonds ] + the 8% interest on [ The amount of money invested in a mutual fund ] = \$1,620 can now be written as

$$0.06x + 0.07y + 0.08z = \$1,620$$

The sentence [ The amount of money invested in municipal bonds ] - [ The amount of money

invested in a mutual fund ] = \$6,000 can now be written as

$$y - z = \$6,000$$

We have converted the problem from one described by words to one that is described by three equations.

$$x + y + z = \$25,000 \quad (1)$$

$$0.06x + 0.07y + 0.08z = \$1,620 \quad (2)$$

$$y - z = \$6,000 \quad (3)$$

#### **SUBSTITUTION:**

The process of substitution involves several steps:

Step 1: Solve for one of the variables in one of the equations. It makes no difference which equation and which variable you choose. Let's solve for y in equation (3) because the equation only has two variables.

$$y - z = \$6,000$$

$$y = \$6,000 + z$$

Step 2: Substitute this value for y in equations (1) and (2). This will change equations (1) and (2) to equations in the two variables x and z. Call the changed equations (4) and (5).

$$x + y + z = \$25,000$$

$$x + (\$6,000 + z) + z = \$25,000$$

$$x + 2z = \$19,000$$

$$0.06x + 0.07y + 0.08z = \$1,620$$

$$0.06x + 0.07(\$6,000 + z) + 0.08z = \$1,620$$

$$0.06x + 0.15z = \$1,200$$

or

$$x + 2z = \$19,000 \quad (4)$$

$$0.06x + 0.15z = \$1,200 \quad (5)$$

Step 3: Solve for  $x$  in equation (4).

$$x + 2z = \$19,000$$

$$x = \$19,000 - 2z$$

Step 4: Substitute this value of  $x$  in equation (5). This will give you an equation in one variable.

$$0.06x + 0.15z = \$1,200$$

$$0.06(\$19,000 - 2z) + 0.15z = \$1,200$$

$$0.03z = 60$$

Step 5: Solve for  $z$ .

$$0.03z = 60$$

$$z = \$2,000$$

Step 6: Substitute this value of  $z$  in equation (4) and solve for  $x$ .

$$\begin{aligned}
 x + 2z &= \$19,000 \\
 x + 2(\$2,000) &= \$19,000 \\
 x &= \$15,000
 \end{aligned}$$

Step 7: Substitute **\$15,000** for **x** and **\$2,000** for **z** in equation (1) and solve for **y**.

$$\begin{aligned}
 \$15,000 + y + \$2,000 &= \$25,000 \\
 y &= \$8,000
 \end{aligned}$$

The solutions: **\$15,000** is invested in the monkey market account, **\$8,000** is invested in the municipal bonds, and **\$2,000** is invested in mutual funds.

Step 8: Check the solutions:

$$\begin{aligned}
 \$15,000 + \$8,000 + \$2,000 &= \$25,000 \rightarrow \text{Yes} \\
 0.06(\$15,000) + 0.07(\$8,000) + 0.08(\$2,000) &= \$1,620 \rightarrow \text{Yes} \\
 \$8,000 - \$2,000 &= \$6,000 \rightarrow \text{Yes}
 \end{aligned}$$

### ELIMINATION:

The process of elimination involves several steps: First you reduce three equations to two equations with two variables, and then to one equation with one variable.

Step 1: Decide which variable you will eliminate. It makes no difference which one you choose. Let us eliminate **x** first because **x** is missing from equation (3).

$$\begin{aligned} (1) \quad x + y + z &= \$25,000 \\ (2) \quad 0.06x + 0.07y + 0.08z &= \$1,620 \\ (3) \quad y - z &= \$6,000 \end{aligned}$$

Step 2: Multiply both sides of equation (1) by **-0.06** and then add the transformed equation (1) to equation (2) to form equation (4).

$$\begin{aligned} & -0.06x - 0.06y - 0.06z = -\$1,500 \\ (1) : & \\ & 0.06x + 0.07y + 0.08z = \$1,620 \\ (2) : & \\ & 0.01y + 0.02z = \$120 \\ (4) : & \end{aligned}$$

Step 3: We now have two equations with two variables.

$$\begin{aligned} & y - z = \$6,000 \\ (3) : & \\ & 0.01y + 0.02z = \$120 \\ (4) : & \end{aligned}$$

Step 4: Multiply both sides of equation (3) by **0.02** and add to equation (4) to create equation (5) with just one variable.

$$\begin{aligned} & 0.02y - 0.02z = \$120 \\ (3) : & \\ & 0.01y + 0.02z = \$120 \\ (4) : & \\ & 0.03y = \$240 \\ (5) : & \end{aligned}$$

Step 5: Solve for **y** in equation (5).

$$\begin{aligned} 0.03y &= \$240 \\ y &= \$8,000 \end{aligned}$$

Step 6: Substitute **\$8,000** for **y** in equation (3) and solve for **z**.

$$y - z = \$6,000$$

$$\$8,000 - z = \$6,000$$

$$z = \$2,000$$

Step 7: Substitute **\$8,000** for **y** and **\$2,000** for **z** in equation (1) and solve for **x**.

$$x + y + z = \$25,000$$

$$x + \$8,000 + \$2,000 = \$25,000$$

$$x = \$15,000$$