

Example 1

Solve this system of equations using elimination.

$$\begin{cases} 4x - 2y + 3z = 1 & (1) \\ x + 3y - 4z = -7 & (2) \\ 3x + y + 2z = 5 & (3) \end{cases}$$

All the equations are already in the required form.

Choose a variable to eliminate, say x , and select two equations with which to eliminate it, say equations (1) and (2).

$$\begin{array}{rcl} 4x - 2y + 3z = 1 & \rightarrow & 4x - 2y + 3z = 1 \\ x + 3y - 4z = -7 & \xrightarrow{\text{multiply } (-4)} & -4x - 12y + 16z = 28 \\ & & \hline & & -14y + 19z = 29 \end{array} \quad (4)$$

Select a different set of two equations, say equations (2) and (3), and eliminate the same variable.

$$\begin{array}{rcl} x + 3y - 4z = -7 & \xrightarrow{\text{multiply } (-3)} & -3x - 9y + 12z = 21 \\ 3x + y + 2z = 5 & \xrightarrow{\text{multiply } (-3)} & 3x + y + 2z = 5 \\ & & \hline & & -8y + 14z = 26 \end{array} \quad (5)$$

Solve the system created by equations (4) and (5).

$$\begin{array}{rcl}
 -14y + 19z = 29 & \xrightarrow{\text{multiply } (-8)} & 112y - 152z = -232 \\
 -8y + 14z = 26 & \xrightarrow{\text{multiply } (14)} & \underline{-112y + 196z = 364} \\
 & & 44z = 132 \\
 & & z = 3
 \end{array}$$

Now, substitute $z = 3$ into equation (4) to find y .

$$\begin{aligned}
 -14y - 19z &= 29 \\
 -14y - 19(3) &= 29 \\
 -14y + 57 &= 29 \\
 -14y &= -28 \\
 y &= 2
 \end{aligned}$$

Use the answers from Step 4 and substitute into any equation involving the remaining variable.

$$\begin{aligned}
 x + 3y - 4z &= -7 \\
 x + 3(2) - 4(3) &= -7 \\
 x + 6 - 12 &= -7
 \end{aligned}$$

Using equation (2), $x = -1$

Check the solution in all three original equations.

$$4x - 2y + 3z = 1$$

$$4(-1) - 2(2) + 3(3) \stackrel{?}{=} 1$$

$$-4 - 4 + 9 \stackrel{?}{=} 1$$

$$1 = 1$$

$$x + 3y - 4z = -7$$

$$-1 + 3(2) - 4(3) \stackrel{?}{=} -7$$

$$-1 + 6 - 12 \stackrel{?}{=} -7$$

$$-7 = -7$$

$$3x + y + 2z = 5$$

$$3(-1) + 2 + 2(3) = 5$$

$$-3 + 2 + 6 = 5$$

$$5 = 5$$

The solution is $x = -1$, $y = 2$, $z = 3$.

Example 2

Solve this system of equations using the elimination method.

$$\begin{cases} x = 3z - 5 & (1) \\ 2x + 2z = y + 16 & (2) \\ 7x - 5z = 3y + 19 & (3) \end{cases}$$

Write all equations in standard form.

$$x - 3z = -5 \quad (1)$$

$$2x - y + 2z = 16 \quad (2)$$

$$7x - 3y - 5z = 19 \quad (3)$$

Notice that equation (1) already has the y eliminated. Therefore, use equations (2) and (3) to eliminate y . Then use this result, together with equation (1), to solve for x and z . Use these results and substitute into either equation (2) or (3) to find y .

$$\begin{array}{rcl} 2x - y + 2z = 16 & \xrightarrow{\text{multiply } (-3)} & -6x + 3y - 6z = -48 \\ 7x - 3y - 5z = 19 & \rightarrow & \underline{7x - 3y - 5z = 19} \\ & & x - 11z = -29 \end{array}$$

$$\begin{array}{rcl} x - 3z = -5 & \xrightarrow{\text{multiply } (-1)} & -x + 3z = 5 \\ x - 11z = -29 & \rightarrow & \underline{x - 11z = -29} \\ & & -8z = -24 \\ & & z = 3 \end{array}$$

Substitute $z = 3$ into equation (1).

$$x - 3z = -5$$

$$x - 3(3) = -5$$

$$x - 9 = -5$$

$$x = 4$$

Substitute $x = 4$ and $z = 3$ into equation (2).

$$2x - y - 2z = 16$$

$$2(4) - y + 2(3) = 16$$

$$8 - y + 6 = 16$$

$$-y = 2$$

$$y = -2$$

Use the original equations to check the solution (the check is left to you).

The solution is $x = 4$, $y = -2$, $z = 3$.