

FRACTION REVIEW

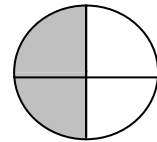
A. INTRODUCTION

1. What is a fraction?

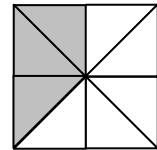
A fraction consists of a numerator (part) on top of a denominator (total) separated by a horizontal line. For example, the fraction of the circle which is shaded is:

2 (parts shaded)
4 (total parts)

Fraction = $\frac{\text{numerator}}{\text{denominator}}$

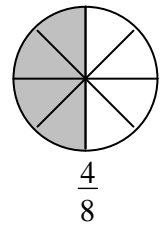
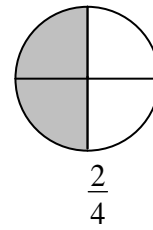
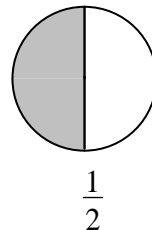


In the square on the right, the fraction shaded is $\frac{3}{8}$ and the fraction unshaded is $\frac{5}{8}$



2. Equivalent Fractions – Multiplying

The three circles on the right each have equal parts shaded, yet are represented by different but equal fractions. These fractions, because they are equal, are called equivalent fractions.



Any fraction can be changed into an equivalent fraction

by multiplying **both** the numerator and denominator by the same number

$$\frac{1}{2} \times 2 = \frac{2}{4}$$

or

$$\frac{1}{2} \times 4 = \frac{4}{8}$$

so

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

Similarly

$$\frac{5}{9} \times 2 = \frac{10}{18}$$

or

$$\frac{5}{9} \times 3 = \frac{15}{27}$$

so

$$\frac{5}{9} = \frac{10}{18} = \frac{15}{27}$$

You can see from the above examples that each fraction has an infinite number of fractions that are equivalent to it.

3. **Equivalent Fractions – Dividing (Reducing)**

Equivalent fractions can also be created if both the numerator and denominator can be divided by the same number (a factor) evenly.

This process is called “reducing a fraction” by dividing a common factor (a number which divides into both the numerator and denominator evenly).

$$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

$$\frac{27}{81} \div \frac{9}{9} = \frac{3}{9}$$

$$\frac{5}{30} \div \frac{5}{5} = \frac{1}{6}$$

$$\frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

4. **Simplifying a Fraction (Reducing to its Lowest Terms)**

It is usual to reduce a fraction until it can't be reduced any further.

A simplified fraction has no common factors which will divide into both numerator and denominator.

Notice that, since 27 and 81 have a common factor of 9, we find that $\frac{3}{9}$ is an equivalent fraction.

But this fraction has a factor of 3 common to both numerator and denominator.

So, we must reduce this fraction again. It is difficult to see, but if we had known that 27 was a factor (divides into both parts of the fraction evenly), we could have arrived at the answer in one step

$$\frac{27}{81} \div \frac{9}{9} = \frac{3}{9}$$

$$\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$$

$$\frac{27}{81} \div \frac{27}{27} = \frac{1}{3}$$

e.g. $\frac{8}{24} \div \frac{8}{8} = \frac{1}{3}$ $\frac{45}{60} \div \frac{15}{15} = \frac{3}{4}$

5. **EXERCISE 1: Introduction to Fractions**

a) Find the missing part of these equivalent fractions

1) $\frac{2}{3} = \frac{\quad}{6}$ 2) $\frac{3}{4} = \frac{\quad}{12}$ 3) $\frac{5}{8} = \frac{\quad}{40}$

4) $\frac{1}{16} = \frac{\quad}{32}$ 5) $\frac{2}{15} = \frac{\quad}{45}$ 6.) $\frac{7}{9} = \frac{\quad}{27}$

7) $\frac{7}{10} = \frac{\quad}{100}$ 8) $\frac{3}{4} = \frac{\quad}{44}$

x 2

Example: $\frac{3}{5} = \frac{\quad}{10}$

x 2

Since $5 \times 2 = 10$,
multiply the numerator
by 2, also.

So, $\frac{3}{5} = \frac{6}{10}$

b) Find the missing part of these equivalent fractions.

1) $\frac{8}{16} = \frac{\quad}{4}$ 2) $\frac{24}{27} = \frac{\quad}{9}$

3) $\frac{6}{10} = \frac{\quad}{5}$ 4) $\frac{25}{35} = \frac{\quad}{7}$

5) $\frac{20}{30} = \frac{\quad}{6}$ 6) $\frac{90}{100} = \frac{\quad}{50}$

÷ 5

Example: $\frac{5}{10} = \frac{\quad}{2}$

÷ 5

Since $10 \div 5 = 2$ divide
the numerator by 5, also.

So, $\frac{5}{10} = \frac{1}{2}$

c) Simplify the following fractions (reduce to lowest terms).

1) $\frac{9}{12}$ 2) $\frac{8}{12}$ 3) $\frac{6}{8}$ 4) $\frac{15}{20}$
 5) $\frac{20}{25}$ 6) $\frac{14}{21}$ 7) $\frac{8}{16}$ 8) $\frac{24}{36}$
 9) $\frac{66}{99}$ 10) $\frac{18}{30}$

B. TYPES OF FRACTIONS

1. Common Fractions

A common fraction is one in which the numerator is less than the denominator (or a fraction which is less than the number 1). A common fraction can also be called a proper fraction.

e.g. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{88}{93}$, $\frac{8}{15}$ are all common fractions.

2. Fractions that are Whole Numbers

Some fractions, when reduced, are really whole numbers (1, 2, 3, 4... etc). Whole numbers occur if the denominator divides into the numerator evenly.

e.g. $\frac{8}{4}$ is the same as $\frac{8}{4} \div \frac{4}{4} = \frac{2}{1}$ or 2
 $\frac{30}{5}$ is the same as $\frac{30}{5} \div \frac{5}{5} = \frac{6}{1}$ or 6

So, the fraction $\frac{30}{5}$ is really the whole number 6.

Notice that a whole number can always be written as a fraction with a denominator of 1.

e.g. $10 = \frac{10}{1}$

3. Mixed Numbers

A mixed number is a combination of a whole number and a common fraction.

e.g. $2\frac{3}{5}$ (two and three-fifths)
 $27\frac{2}{9}$ (twenty-seven and two-ninths)
 $9\frac{3}{6} = 9\frac{1}{2}$ (always reduce fractions)

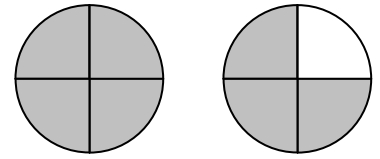
4. Improper Fractions

An improper fraction is one in which the numerator is larger than the denominator.

From the circles on the right, we see that $1\frac{3}{4}$ (mixed number)

is the same as $\frac{7}{4}$ (improper fraction).

An improper fraction, like $\frac{7}{4}$, can be changed to a mixed number by dividing the denominator into the numerator and expressing the remainder (3) as the numerator.



$$1\frac{3}{4} = \frac{7}{4}$$

$$\frac{7}{4} = 4 \overline{)7} = 1\frac{3}{4}$$

e.g. $\frac{16}{5} = 3\frac{1}{5}$ $\frac{29}{8} = 3\frac{5}{8}$ $\frac{14}{3} = 4\frac{2}{3}$

A mixed number can be changed to an improper fraction by changing the whole number to a fraction with the same denominator as the common fraction.

$$2\frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$$

$$10\frac{1}{9} = \frac{90}{9} + \frac{1}{9} = \frac{91}{9}$$

A simple way to do this is to multiply the whole number by the denominator, and then add the numerator.

e.g. $4\frac{5}{9} = \frac{4 \times 9 + 5}{9} = \frac{36 + 5}{9} = \frac{41}{9}$
 $10\frac{2}{7} = \frac{10 \times 7 + 2}{7} = \frac{70 + 2}{7} = \frac{72}{7}$

5. Simplifying fractions

All types of fractions must always be simplified (reduced to lowest terms).

e.g. $\frac{6}{9} = \frac{2}{3}$, $2\frac{5}{25} = 2\frac{1}{5}$, $\frac{27}{18} = \frac{3}{2} = 1\frac{1}{2}$

Note that many fractions can not be reduced since they have no common factors.

e.g. $\frac{17}{21}$, $\frac{4}{9}$, $\frac{18}{37}$

6. **EXERCISE 2 : Types of Fractions**

a) Which of the following are common fractions (C), whole numbers (W), mixed numbers (M) or improper fractions (I)?

- 1) $\frac{2}{3}$ 2) $3\frac{4}{5}$ 3) $\frac{7}{5}$ 4) $\frac{8}{8}$ 5) $\frac{24}{2}$
6) $5\frac{8}{19}$ 7) $2\frac{3}{3}$ 8) $\frac{25}{24}$ 9) $\frac{24}{25}$ 10) $\frac{12}{12}$

b) Change the following to mixed numbers:

- 1) $\frac{7}{5}$ 2) $\frac{18}{11}$ 3) $\frac{70}{61}$ 4) $\frac{12}{5}$ 5) $\frac{100}{99}$ 6) $\frac{25}{2}$

c) Change the following to improper fractions:

- 1) $2\frac{1}{5}$ 2) $6\frac{3}{8}$ 3) $8\frac{2}{3}$ 4) $11\frac{1}{5}$ 5) $9\frac{4}{5}$ 6) $4\frac{3}{4}$

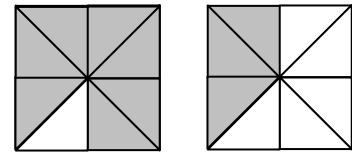
d) Simplify the following fractions:

- 1) $\frac{28}{40}$ 2) $\frac{80}{10}$ 3) $2\frac{12}{18}$ 4) $5\frac{27}{54}$ 5) $\frac{25}{15}$ 6) $\frac{90}{12}$

C. COMPARING FRACTIONS

In the diagram on the right, it is easy to see that $\frac{7}{8}$ is larger than $\frac{3}{8}$ (since 7 is larger than 3).

However, it is not as easy to tell that $\frac{7}{8}$ is larger than $\frac{5}{6}$.



$$\frac{7}{8} = \frac{3}{8}$$

In order to compare fractions, we must have the same (common)

denominators. This process is called

“Finding the Least Common Denominator” and is usually

abbreviated as finding the LCD or LCM (lowest common multiple).

Which is larger: $\frac{7}{8}$ or $\frac{5}{6}$?

In order to compare these fractions, we must change both fractions to equivalent fractions with a common denominator.

To do this, take the largest denominator (8) and examine multiples of it, until the other denominator (6) divides into it.

Notice that, when we multiply 8×3 , we get 24, which 6 divides into.

Now change the fractions to 24th s.

When we change these fractions to equivalent fractions with an LCD of 24, we can easily see

that $\frac{7}{8}$ is larger than $\frac{5}{6}$ since $\frac{21}{24}$ is greater than $\frac{20}{24}$.

$$8 \times 1 = 8$$

(6 doesn't divide into 8)

$$8 \times 2 = 16$$

(6 doesn't divide into 16)

$$8 \times 3 = 24 \quad \text{LCD}$$

$\begin{array}{r} x 3 \\ x 3 \end{array}$	$\begin{array}{r} x 4 \\ x 4 \end{array}$
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$$\frac{7}{8} = \frac{21}{24} \quad \frac{5}{6} = \frac{20}{24}$$

Which is larger: $\frac{4}{9}$ or $\frac{5}{12}$?

Examine multiples of the larger denominator (12) until the smaller denominator divides into it. This tells us that the LCD is 36.
 Now, we change each fraction to equivalent fractions with the LCD of 36.

$12 \times 1 = 12$
 $12 \times 2 = 24$
 $12 \times 3 = 36$ (LCD)
 LCD is 36.

$$\begin{array}{r} \text{x4} \\ \frac{4}{9} = \frac{16}{36} \\ \text{x4} \end{array}$$

$$\begin{array}{r} \text{x3} \\ \frac{5}{12} = \frac{15}{36} \\ \text{x3} \end{array}$$

So, $\frac{4}{9}$ is larger than $\frac{5}{12}$

Which is larger: $\frac{4}{5}$ or $\frac{13}{15}$ or $\frac{11}{12}$?

Find the LCD by examining multiples of 15. Notice that, when we multiply 15×4 , we find that 60 is the number that all denominators divide into.

$15 \times 1 = 15$
 $15 \times 2 = 30$
 $15 \times 3 = 45$
 $15 \times 4 = 60$ (LCD)

$$\begin{array}{r} \text{x12} \\ \frac{4}{5} = \frac{48}{60} \\ \text{x12} \end{array}$$

$$\begin{array}{r} \text{x4} \\ \frac{13}{15} = \frac{52}{60} \\ \text{x4} \end{array}$$

$$\begin{array}{r} \text{x5} \\ \frac{11}{12} = \frac{55}{60} \\ \text{x5} \end{array}$$

So, $\frac{11}{12}$ is the largest fraction.

Which is larger: $\frac{7}{9}$ or $\frac{13}{18}$?

Notice that one denominator (9) divides into the other denominator (18). This means that the LCD = 18 and we only have to change one fraction $\left(\frac{7}{9}\right)$ to an equivalent fraction.

$$\begin{array}{ccc} & \times 2 & \\ \frac{7}{9} & = & \frac{14}{18} \\ & \times 2 & \end{array} \qquad \text{So, } \frac{7}{9} \text{ is larger than } \frac{13}{18}$$



1. **EXERCISE 3: Comparing Fractions**

Which is the largest fraction? (find LCD first)

1) $\frac{7}{13}$ or $\frac{6}{13}$

2) $1\frac{9}{10}$ or $\frac{18}{10}$

3) $\frac{4}{5}$ or $\frac{9}{10}$

4) $\frac{3}{13}$ or $\frac{5}{12}$

5) $\frac{5}{8}$ or $\frac{4}{7}$

6) $\frac{1}{2}$ or $\frac{6}{11}$ or $\frac{7}{12}$

7) $\frac{2}{3}$ or $\frac{11}{15}$

8) $\frac{4}{9}$ or $\frac{5}{12}$ or $\frac{3}{8}$

9) $\frac{1}{4}$ or $\frac{3}{16}$

D. ADDING FRACTIONS

There are four main operations that we can do with numbers: addition (+), subtraction (-), multiplication (x), and division (÷).

In order to add or subtract, fractions must have common denominators.

This is not required for multiplication or division.

1. Adding with Common Denominators

To add fractions, if the denominators are the same, we simply add the numerators and keep the same denominators.

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

e.g.

Add $\frac{1}{12}$ and $\frac{5}{12}$

Since the denominators are common, simply add the numerators. Notice that we must reduce the answer, if possible.

$$\begin{aligned}\frac{1}{12} + \frac{5}{12} &= \frac{6}{12} \\ &= \frac{1}{2}\end{aligned}$$

2. Adding When One Denominator is a Multiple of the Other

Add $\frac{2}{9}$ and $\frac{5}{27}$

$$\begin{array}{r} \frac{2}{9} = \frac{6}{27} \\ \text{x3} \\ \text{x3} \end{array}$$

Notice that the denominators are not common. Also notice that 27 is a multiple of 9 (since $9 \times 3 = 27$). This means that the LCD = 27 (see the last example in “Comparing Fractions”).

$$\begin{aligned}\frac{2}{9} + \frac{5}{27} &= \frac{6}{27} + \frac{5}{27} \\ &= \frac{11}{27}\end{aligned}$$

3. Adding Any Fraction

Add $\frac{7}{12}$ and $\frac{13}{15}$

$$\begin{aligned} \frac{7}{12} + \frac{13}{15} &= \frac{35}{60} + \frac{52}{60} \\ &= \frac{87}{60} \\ &= 1\frac{9}{20} \end{aligned}$$

We must find a common denominator by examining multiples of the largest denominator. We find that the LCD = 60.

Add $1\frac{5}{6}$ and $2\frac{3}{8}$

When adding mixed numbers, add the whole numbers and the fractions separately. Find common denominators and add.

$$\begin{array}{r} 1\frac{5}{6} = 1\frac{20}{24} \\ + 2\frac{3}{8} = 2\frac{9}{24} \\ \hline \end{array}$$

If an improper fraction occurs in the answer, change it to a common fraction by doing the following.

$$\begin{array}{r} \text{total equals } 3\frac{29}{24} \\ 3\frac{29}{24} = 3 + 1\frac{5}{24} \\ = 4\frac{5}{24} \end{array}$$

4. The Language of Addition

$\frac{1}{2} + \frac{2}{3}$ CAN BE WORDED

$\frac{1}{2}$ plus $\frac{2}{3}$

$\frac{1}{2}$ and $\frac{2}{3}$

total of $\frac{1}{2}$ and $\frac{2}{3}$

sum of $\frac{1}{2}$ and $\frac{2}{3}$

addition of $\frac{1}{2}$ and $\frac{2}{3}$

$\frac{1}{2}$ combined with $\frac{2}{3}$

$\frac{1}{2}$ more than (or greater than) $\frac{2}{3}$

Note: All of these can be worded with the fractions in reverse order:

e.g. $\frac{2}{3}$ plus $\frac{1}{2}$ *is the same as* $\frac{1}{2}$ plus $\frac{2}{3}$

5. **EXERCISE 4: Adding Fractions**

a) Add the following:

1) $\frac{1}{5} + \frac{2}{5}$

2) $\frac{4}{5} + \frac{3}{5}$

3) $\frac{4}{9} + \frac{2}{9}$

4) $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

b) Find the sum of:

1) $\frac{2}{3} + \frac{1}{9}$

2) $\frac{1}{2} + \frac{3}{8}$

3) $\frac{1}{4} + \frac{5}{16}$

4) $\frac{2}{3} + \frac{4}{15}$

c) Add the following:

1) $3\frac{1}{2} + 4\frac{1}{4}$

2) $9\frac{2}{3} + 3\frac{1}{6}$

3) $8\frac{1}{2} + 4\frac{4}{5}$

4) $2\frac{3}{4} + 6\frac{1}{2}$

5) $4\frac{1}{3} + 6\frac{5}{6}$

6) $6\frac{1}{3} + 8\frac{3}{4}$

7) $7\frac{2}{3} + \frac{4}{5}$

8) $8\frac{2}{3} + 6\frac{1}{4} + 1\frac{3}{8}$

d) Evaluate the following:

1) $\frac{2}{3}$ and $\frac{3}{7}$

2) total of $\frac{5}{6}$ and $\frac{3}{8}$

3) $\frac{1}{2}$ plus $\frac{1}{5}$

4) $\frac{3}{2}$ greater than $\frac{5}{7}$

5) $\frac{5}{12}$ combined with $\frac{3}{8}$

6) sum of $\frac{1}{6}$ and $\frac{3}{14}$

E. SUBTRACTING FRACTIONS

1. Common Fractions

As in addition, we must have common denominators in order to subtract. Find the LCD; change the fractions to equivalent fraction with the LCD as the denominator. Then subtract the numerators, but keep the same denominator.

$$\begin{aligned}\frac{5}{8} - \frac{3}{8} &= \frac{2}{8} \text{ or } \frac{1}{4} \\ \frac{2}{3} - \frac{3}{8} &= \frac{16}{24} - \frac{9}{24} \\ &= \frac{7}{24}\end{aligned}$$

2. Mixed Numbers

When subtracting whole numbers, subtract the whole numbers, and then subtract the fractions separately.

$$3\frac{5}{9} - 1\frac{3}{9} = 2\frac{2}{9}$$

However, if the common fraction we are subtracting is smaller than the other common fraction, we must borrow the number “1” from the large whole number.

i.e. $4\frac{2}{7} = 3 + \frac{7}{7} + \frac{2}{7}$, or $3\frac{9}{7}$

$$\begin{aligned}4\frac{2}{7} - 2\frac{5}{7} &= 3\frac{9}{7} - 2\frac{5}{7} \\ &= 1\frac{4}{7}\end{aligned}$$

To subtract $1\frac{3}{4}$ **from** $6\frac{2}{3}$, first change the common fractions to equivalent fractions with the LCD. Since $\frac{8}{12}$ is smaller than $\frac{9}{12}$, borrow from 6.

$$6\frac{8}{12} = 5\frac{12}{12} + \frac{8}{12} = 5\frac{20}{12}$$

$$\begin{aligned}6\frac{2}{3} - 1\frac{3}{4} &= 6\frac{8}{12} - 1\frac{9}{12} \\ &= 5\frac{20}{12} - 1\frac{9}{12} \\ &= 4\frac{11}{12}\end{aligned}$$

3. The Language of Subtraction

$\frac{5}{6} - \frac{2}{3}$ CAN BE WORDED

$$\frac{5}{6} \text{ minus } \frac{2}{3} \text{ (NOT } \frac{2}{3} \text{ minus)}$$

$$\frac{2}{3} \text{ subtracted from } \frac{5}{6}$$

$$\frac{2}{3} \text{ from } \frac{5}{6}$$

$$\frac{2}{3} \text{ less than } \frac{5}{6}$$

$$\frac{5}{6} \text{ decreased by or lowered by } \frac{2}{3}$$

$$\text{the difference of } \frac{5}{6} \text{ and } \frac{2}{3}$$

NOTE: Unlike addition, we can not reword the above with the fractions in reverse order:

i.e. $\frac{1}{2} - \frac{2}{3}$ is **NOT** the same as $\frac{2}{3} - \frac{1}{2}$

4. EXERCISE 5: Subtracting Fractions

a) Subtract the following:

1) $\frac{9}{12} - \frac{1}{8}$

2) $\frac{14}{15} - \frac{1}{6}$

3) $\frac{5}{6} - \frac{3}{8}$

4) $\frac{7}{9} - \frac{2}{3}$

5) $9\frac{2}{3} - 6\frac{1}{6}$

6) $4\frac{1}{2} - 1\frac{1}{4}$

7) $\frac{3}{4} - \frac{5}{8}$

8) $\frac{11}{12} - \frac{2}{3}$

b) Subtract the following:

1) $6\frac{1}{3} - 2\frac{2}{3}$

2) $13\frac{1}{4} - 5\frac{3}{4}$

3) $5\frac{5}{7} - 4\frac{6}{7}$

4) $4\frac{3}{4} - 1\frac{11}{12}$

5) $16\frac{2}{3} - 5\frac{3}{4}$

6) $9\frac{1}{6} - 4\frac{3}{8}$

c) Find the following:

1) What is $\frac{5}{8}$ minus $\frac{3}{16}$?

2) $\frac{2}{7}$ decreased by $\frac{1}{21}$ is what?

3) What is $\frac{4}{9}$ less than $\frac{7}{9}$?

4) What is $\frac{1}{6}$ from $\frac{9}{24}$?

F. MULTIPLYING FRACTIONS

1. Common Fractions

When multiplying fractions, a common denominator is not needed. Simply multiply the numerators and multiply the denominators separately.

$$\frac{2}{3} \times \frac{5}{9} = \frac{2 \times 5}{3 \times 9} = \frac{10}{27}$$

Sometimes, we can reduce the fractions before multiplying.

$$\frac{3}{5} \times \frac{5}{7} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times 7} = \frac{3}{7}$$

Any common factor in either numerator can cancel with the same factor in the denominator. Multiply after cancelling (reducing).

$$\frac{4}{9} \times \frac{3}{8} = \frac{\overset{1}{\cancel{4}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{8}}} = \frac{1}{6}$$

Note that any whole number (16) has the number “1” understood in its denominator.

$$\frac{5}{8} \times 16 = \frac{5 \times \overset{2}{\cancel{16}}}{\underset{1}{\cancel{8}} \times 1} = 10$$

If more than two fractions are multiplied, the same principles apply.

$$\frac{2}{9} \times \frac{3}{20} \times \frac{1}{4} = \frac{2 \times 3 \times 1}{9 \times 20 \times 4} = \frac{1}{120}$$

2. Mixed Numbers

Mixed numbers must be changed to improper fractions before multiplying. Remember that a mixed number (like $2\frac{3}{4}$) can be changed to an improper fraction by multiplying the whole number (2) by the denominator (4) and then adding the numerator.

$$2\frac{3}{4} \times 4\frac{8}{11} = \frac{11}{4} \times \frac{52}{11} = 13$$

See page 6 for instructions on changing a mixed number to an improper fraction.

$$\begin{aligned} 5\frac{4}{9} \times 2\frac{3}{7} &= \frac{49}{9} \times \frac{17}{7} \\ &= \frac{119}{9} \\ &\text{or } 13\frac{2}{9} \end{aligned}$$

3. The Language of Multiplication

$\frac{1}{2} \times \frac{2}{3}$ CAN BE WORDED
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$$\frac{1}{2} \text{ multiplied by } \frac{2}{3}$$

$$\frac{1}{2} \text{ by } \frac{2}{3}$$

$$\frac{1}{2} \text{ of } \frac{2}{3}$$

$$\text{the product of } \frac{1}{2} \text{ and } \frac{2}{3}$$

NOTE: When multiplying, it doesn't matter which fraction is first.

i.e. $\frac{1}{2} \times \frac{2}{3}$ is the same as $\frac{2}{3} \times \frac{1}{2}$

4. EXERCISE 6: Multiplying Fractions

a) Multiply (cancel first, when possible):

1) $\frac{2}{3} \times \frac{3}{4}$

2) $\frac{8}{9} \times \frac{12}{16}$

3) $\frac{5}{3} \times \frac{9}{15}$

4) $\frac{2}{5} \times \frac{15}{21}$

5) $\frac{5}{8} \times \frac{48}{125}$

6) $\frac{3}{4} \times \frac{16}{27} \times \frac{9}{16}$

7) $\frac{5}{3} \times \frac{7}{4} \times 90 \times 20 \times \frac{16}{14}$

8) $2\frac{4}{5} \times 3\frac{4}{7}$

9) $8\frac{2}{3} \times \frac{15}{21}$

10) $\frac{18}{19} \times \frac{57}{4} \times \frac{6}{32} \times \frac{8}{9} \times \frac{2}{3}$

11) $1\frac{12}{25} \times 2\frac{1}{37}$

12) $2\frac{5}{12} \times 6$

b) Find the following:

1) What is $\frac{2}{3}$ of 45?

2) $\frac{41}{7}$ by 14 is what number?

3) What is $\frac{5}{9}$ times $\frac{27}{10}$?

4) $\frac{4}{5}$ of $2\frac{1}{7}$ is what number?

5) $\frac{1}{2}$ of $\frac{62}{63}$ is what number?

G. DIVIDING FRACTIONS

To divide fractions, we invert (take the reciprocal of) the fraction that we are dividing by, then cancel (reduce), and then multiply.

Taking the reciprocal of a fraction involves “flipping” the fraction so that the numerator and denominator switch places.

Note that a whole number is really a fraction (e.g. $4 = \frac{4}{1}$).

$$\frac{2}{3} \text{ reciprocal } \frac{3}{2}$$

$$\frac{8}{19} \text{ reciprocal } \frac{19}{8}$$

$$4 \text{ reciprocal } \frac{4}{1}$$

1. Common Fractions

Simply invert (take the reciprocal of) the fractions that we are dividing by ($\frac{8}{9}$). Then cancel and multiply.

Note: you can only cancel **after** the division is changed to a multiplication.

$$\frac{5}{7} \div \frac{8}{9} = \frac{5}{7} \times \frac{9}{8} = \frac{45}{56}$$

$$\frac{9}{16} \div \frac{3}{32} = \frac{9}{16} \times \frac{32}{3} = 6$$

2. Mixed Numbers

As in multiplication, mixed numbers must be changed to improper fractions.

$$\begin{aligned} 1\frac{2}{3} \div 2\frac{1}{7} &= \frac{5}{3} \div \frac{15}{7} \\ &= \frac{5}{3} \times \frac{7}{15} \\ &= \frac{7}{9} \end{aligned}$$

3. The Language of Division

$$\frac{1}{2} \div \frac{2}{3} \text{ CAN BE WORDED}$$

$$\frac{1}{2} \text{ divided by } \frac{2}{3}$$

$$\frac{2}{3} \text{ into } \frac{1}{2}$$

$$\text{divide } \frac{1}{2} \text{ by } \frac{2}{3}$$

NOTE: In multiplication, the order of the fractions was not important.

i.e. $\frac{1}{2} \times \frac{2}{3}$ **is the same as** $\frac{2}{3} \times \frac{1}{2}$

In division, this is not the case. The order of the fractions is important.
Consider the following:

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

but $\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}$

4. EXERCISE 7: Dividing Fractions

a) Divide:

1) $\frac{3}{5} \div \frac{9}{15}$

2) $\frac{3}{7} \div \frac{12}{5}$

3) $6 \div \frac{2}{3}$

4) $\frac{7}{12} \div 14$

5) $\frac{13}{15} \div \frac{39}{40}$

6) $3\frac{3}{10} \div \frac{11}{15}$

7) $\frac{27}{30} \div 8\frac{1}{3}$

8) $1\frac{7}{9} \div 1\frac{2}{3}$

9) $4\frac{2}{3} \div 3\frac{1}{2}$

10) $2\frac{3}{11} \div 3\frac{9}{22}$

b) Find the following:

1) $\frac{4}{5}$ divided by $\frac{2}{5}$

2) $\frac{2}{5}$ divided by $\frac{4}{5}$

3) $\frac{9}{4}$ into $\frac{12}{25}$

4) Divide $1\frac{2}{3}$ by $\frac{2}{3}$

5) $\frac{4}{9}$ into $1\frac{4}{5}$

6) Divide $\frac{1}{4}$ by $\frac{2}{5}$

FRACTION REVIEW:

Decide which operation (+, -, x, ÷) by the wording in the question. Then find the answer.

1) What is $\frac{3}{5}$ of 40?

2) How much is $4\frac{2}{3}$ from $6\frac{1}{5}$?

3) How much is $\frac{1}{2}$ from $\frac{2}{3}$?

4) How much is $\frac{3}{7}$ of 21?

5) $\frac{4}{5}$ and $\frac{2}{3}$ equals....?

6) $\frac{2}{3}$ divided by 14 is what number?

7) What is $\frac{9}{10}$ of 300?

8) What is $\frac{2}{5}$ into 12?

9) $\frac{3}{4}$ from $\frac{8}{9}$ equals....?

10) What is $1\frac{2}{3}$ by $3\frac{3}{8}$?

11) How much is $\frac{5}{7}$ of $\frac{21}{50}$?

12) Find the total of $\frac{2}{3}$ and $\frac{1}{6}$ and $\frac{4}{9}$?

13) $\frac{3}{4}$ of $\frac{2}{3}$ equals....?

14) What is $\frac{2}{5}$ greater than $4\frac{3}{5}$?

ANSWER KEY – FRACTION REVIEW

EXERCISE 1: Introduction to Fractions (Page 3)

- a) 1) $\frac{4}{6}$ 2) $\frac{9}{12}$ 3) $\frac{25}{40}$ 4) $\frac{2}{32}$ 5) $\frac{6}{45}$ 6) $\frac{21}{27}$ 7) $\frac{70}{100}$ 8) $\frac{33}{44}$
- b) 1) $\frac{2}{4}$ 2) $\frac{8}{9}$ 3) $\frac{3}{5}$ 4) $\frac{5}{7}$ 5) $\frac{4}{6}$ 6) $\frac{45}{50}$
- c) 1) $\frac{3}{4}$ 2) $\frac{2}{3}$ 3) $\frac{3}{4}$ 4) $\frac{3}{4}$ 5) $\frac{4}{5}$ 6) $\frac{2}{3}$ 7) $\frac{1}{2}$ 8) $\frac{2}{3}$ 9) $\frac{2}{3}$ 10) $\frac{3}{5}$

EXERCISE 2: Types of Fractions (Page 6)

- a) 1) C 2) M 3) I 4) W 5) W 6) M 7) W 8) I 9) C 10) W
- b) 1) $1\frac{2}{5}$ 2) $1\frac{7}{11}$ 3) $1\frac{9}{61}$ 4) $2\frac{2}{5}$ 5) $1\frac{1}{99}$ 6) $12\frac{1}{2}$
- c) 1) $\frac{11}{5}$ 2) $\frac{51}{8}$ 3) $\frac{26}{3}$ 4) $\frac{56}{5}$ 5) $\frac{49}{5}$ 6) $\frac{19}{4}$
- d) 1) $\frac{7}{10}$ 2) 8 3) $2\frac{2}{3}$ 4) $5\frac{1}{2}$ 5) $\frac{5}{3}$ or $1\frac{2}{3}$ 6) $\frac{15}{2}$ or $7\frac{1}{2}$

EXERCISE 3: Comparing Fractions (Page 9)

- 1) $\frac{7}{13}$ 2) $1\frac{9}{10}$ 3) $\frac{9}{10}$ 4) $\frac{5}{12}$ 5) $\frac{5}{8}$ 6) $\frac{7}{12}$ 7) $\frac{11}{15}$ 8) $\frac{4}{9}$ 9) $\frac{1}{4}$

EXERCISE 4: Adding Fractions (Page 12)

- a) 1) $\frac{3}{5}$ 2) $\frac{7}{5}$ or $1\frac{2}{5}$ 3) $\frac{2}{3}$ 4) $\frac{9}{4}$ or $2\frac{1}{4}$
- b) 1) $\frac{7}{9}$ 2) $\frac{7}{8}$ 3) $\frac{9}{16}$ 4) $\frac{14}{15}$
- c) 1) $7\frac{3}{4}$ 2) $12\frac{5}{6}$ 3) $13\frac{3}{10}$ 4) $9\frac{1}{4}$ 5) $11\frac{1}{6}$ 6) $15\frac{1}{12}$ 7) $8\frac{7}{15}$ 8) $16\frac{7}{24}$
- d) 1) $1\frac{2}{21}$ 2) $1\frac{5}{24}$ 3) $\frac{7}{10}$ 4) $2\frac{3}{14}$ 5) $\frac{19}{24}$ 6) $\frac{8}{21}$

EXERCISE 5: Subtracting Fractions (Page 14)

- a) 1) $\frac{5}{8}$ 2) $\frac{23}{30}$ 3) $\frac{11}{24}$ 4) $\frac{1}{9}$ 5) $3\frac{1}{2}$ 6) $3\frac{1}{4}$ 7) $\frac{1}{8}$ 8) $\frac{1}{4}$
- b) 1) $3\frac{2}{3}$ 2) $7\frac{1}{2}$ 3) $\frac{6}{7}$ 4) $2\frac{5}{6}$ 5) $10\frac{11}{12}$ 6) $4\frac{19}{24}$
- c) 1) $\frac{7}{16}$ 2) $\frac{5}{21}$ 3) $\frac{1}{3}$ 4) $\frac{5}{24}$

EXERCISE 6: Multiplying Fractions (Page 16)

- a) 1) $\frac{1}{2}$ 2) $\frac{2}{3}$ 3) 1 4) $\frac{2}{7}$ 5) $\frac{6}{25}$ 6) $\frac{1}{4}$ 7) 6000
- 8) 10 9) $\frac{130}{21}$ or $6\frac{4}{21}$ 10) $1\frac{1}{2}$ 11) 3 12) $\frac{29}{2}$ or $14\frac{1}{2}$
- b) 1) 30 2) 82 3) $\frac{3}{2}$ or $1\frac{1}{2}$ 4) $\frac{12}{7}$ or $1\frac{5}{7}$ 5) $\frac{31}{63}$

EXERCISE 7: Dividing Fractions (Page 18)

- a) 1) 1 2) $\frac{5}{28}$ 3) 9 4) $\frac{1}{24}$ 5) $\frac{8}{9}$ 6) $4\frac{1}{2}$ 7) $\frac{27}{250}$
- 8) $1\frac{1}{15}$ 9) $1\frac{1}{3}$ 10) $\frac{2}{3}$
- b) 1) 2 2) $\frac{1}{2}$ 3) $\frac{16}{75}$ 4) $2\frac{1}{2}$ 5) $4\frac{1}{20}$ 6) $\frac{5}{8}$

Fraction Review (Page 19)

- a) 1) 24 2) $1\frac{8}{15}$ 3) $\frac{1}{6}$ 4) 9 5) $1\frac{7}{15}$ 6) $\frac{1}{21}$ 7) 270
- 8) 30 9) $\frac{5}{36}$ 10) $5\frac{5}{8}$ 11) $\frac{3}{10}$ 12) $1\frac{5}{18}$ 13) $\frac{1}{2}$ 14) 5